Homework 8

PHYS798C Spring 2024
Due Thursday, 11 April, 2024

1 Current-Tuned Kinetic Inductance

(a) Consider a filamentary superconducting wire of cross sectional area σ carrying a time-dependent current $I(t)$. The supercurrent is accelerated, requiring an electric field E that is related to dI/dt through the kinetic inductance: $E = L_k \frac{dI}{dt}$. Use the first London equation to find the following expression for the kinetic inductance, $L_k = \frac{-m^*}{e^*}\frac{dv_s}{dI}$, where v_s is the superfluid velocity.

(b) Using the result from Ginzburg-Landau theory for the current in a filamentary superconductor, $I = e^*\sigma|\psi_\infty|^2 \left(1-(v_s/v_c)^2\right) v_s$ show the the kinetic inductance is given by $L_k = \frac{L_k(0)}{\left(1-(v_s/v_m)^2\right)}$ where $L_k(0) = \mu_0\lambda^2/\sigma$, and $v_c = \sqrt{3}v_m$, and v_m is the maximum speed of the superfluid when the current peaks as a function of v_s.

(c) Now consider a microstrip transmission line made up of two superconducting films having a thickness $t << \lambda$. The above expression for L_k applies here as well. The phase velocity of a travelling electromagnetic wave on the two-conductor transmission line is $v_{ph} = \frac{1}{\sqrt{LC}}$, where $L = L_{mag} + L_k$ is the total inductance, L_{mag} is the magnetic inductance of the films, and C is the capacitance between the films. Write an expression for the phase velocity of the waves as a function of superfluid velocity. In the limit where $L_k(0) \gg L_{mag}$ this can have a strongly tunable wave speed for microwave signals.

2 Little-Parks Experiment and Flux Quantization

Consider a thin superconducting film of thickness $d << \lambda$ deposited onto a cylindrical dielectric filament (like a human hair for example). The radius of the filament's cross section is R. At room temperature, the filament is placed in a longitudinal magnetic field and then cooled down to a temperature below T_c. Then the external field is switched off.

(a) Use fluxoid quantization in the thin cylindrical film to show that $m^* v_s(2\pi R) + q^*\Phi = nh$, where Φ is the trapped flux, n is a positive or negative integer or zero, and h is Planck's constant.

(b) Solve for the superfluid velocity v_s in terms of Φ/Φ_0 (recall that $\Phi_0 = h/e^*$). Plot v_s^2 vs. Φ/Φ_0 for 5 different choices of n centered on $n = 0$. This is effectively the kinetic energy of the supercurrent flow as a function of Φ/Φ_0.

(c) To minimize the kinetic energy as a function of Φ/Φ_0, the superconductor will choose different values of n (i.e. change the number of trapped flux) as the flux changes. This involves creating phase slips by suppressing T_c and allowing n to change by ± 1.

Recall that the superconducting order parameter varies with v_s as $|\psi|^2 = |\psi_\infty|^2 \left(1 - (\frac{m^* \xi_{GL} v_s}{\hbar})^2\right)$. Show that the order parameter will be suppressed to zero when $\frac{1}{\xi_{GL}^2(T^*)} = \left(\frac{m^* v_s}{\hbar}\right)^2$, where T^* represents the suppresed T_c created by the screening supercurrent. Use the definition of the temperature-dependent GL coherence length $\xi_{GL}(T^*) = \xi_{GL}(0)/\sqrt{1-t^*}$, where $t^* = T^*/T_c$, to solve for (and plot) $\Delta T_c/T_c \equiv \frac{T^*-T_c}{T_c}$ in terms of $(n - \Phi/\Phi_0)^2$. These periodic variations in T_c vs. Φ/Φ_0 are measured in the Little-Parks experiment. *Hint*: see "The archived Little-Parks experiment lecture" under Lecture 18 in the Sup. Mat. on the class website.